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Modeling lactation curve in jersey crossbred cows

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Abstract
The present study was conducted to fit lactation curve models for Jersey crossbred dairy cattle in organised dairy farms, so as to find the best fitting model. Lactation curve modeling of Jersey crossbred cows was carried out using daily milk yield data of 259 lactations at two farms of Tamil Nadu Veterinary and Animal Sciences University. Various models were fitted to find the best model of fit for prediction of milk yield, based on four diagnostic parameters *viz.* coefficient of determination (R^2), adjusted R^2, Root Mean Square Error (RMSE) and Durbin-Watson (DW) coefficient. Highest rank obtained by Friedman's test was chosen as best model. The results of the study revealed that the highest value of Coefficient of Determination (R^2 - 0.74) was obtained in both Ali and Schaeffer and Cubic models, while the lowest R^2 (0.45) was in Inverse Quadratic Polynomial modified model. Average Root Mean square Error was the lowest in Inverse Quadratic Polynomial modified model (0.08 kg) and highest was in Quadratic model. Durbin-Watson coefficients in different models ranged from 0.76 (Inverse Quadratic Polynomial modified model) to 1.09 (Ali and Schaeffer model), indicating positive auto-correlation of residuals for the models considered. Ali and Schaeffer, Cubic, Quadratic cum log, Quadratic and Mixed log models were found to the best lactation curve models for Jersey crossbred cows in that order. Lactation parameters estimated through this study can be used in genetic evaluations and in selection of cows to improve persistency and yield of milk production.

Keywords: Jersey crossbred cows – lactation curve – modeling

1. Introduction
Lactation curve is the graphical representation of milk yield against time. A typical lactation curve has two characteristic parts - a rapid increase from calving to peak period in early stage of lactation and a gradual decline from peak yield to the end of lactation [1]. The advantages of modeling the lactation curve are that the milk yield of a cow in a lactation can be predicted with minimum error, it can be used in the process of cow/sire evaluation and it can thus enable an anticipated choice of animals to be culled [2]. It can also help in predicting expected missing values on field records, besides giving a concise summary of biological efficiency and persistency of a dairy cow. Further, the knowledge of lactation curves in dairy cattle is important for decisions on herd management and selection strategies, as they can be a key element in determining optimum strategies for insemination, replacement and genetic evaluation for improvement of milk production traits [3]. With this background, the present study was conducted with the objective of fitting different lactation curve models for Jersey crossbred cows in organized farms of Tamil Nadu Veterinary and Animal Sciences University (TANUVAS).

2. Materials and Methods
Daily milk yield data on 259 lactations of Jersey crossbred cows were collected for a period of eleven years (2005-2015) from the records maintained at two farms of Tamil Nadu Veterinary and Animal Sciences University (TANUVAS), *viz.* Post Graduate Research Institute in Animal Sciences (PGRIAS), Kattupakkam and University Research Farm (URF), Chennai. Data so collected were subjected to descriptive and exploratory analysis. The above data were collected during the period from February 2016 to May 2016. Individual test day yield was regressed on the day of lactation by the statistical models using IBM SPSS Statistics 20.0. Various lactation curve models were fitted in the study, as detailed below, for assessing their ability to explain the lactation and lactation curve in Jersey crossbred cows

Name Of The Model	Model Description
Exponential decline function (EDF) [4]	$Y_t = ae^{-ct}$; $\ln(Y_t) = \ln(a) - ct + e$
Parabolic exponential model (PEM) [5]	$Y_t = a\exp(bt - ct^2)$; $\ln(Y_t) = \ln(a) + bt - ct^2 + e$
Inverse quadratic polynomial (IQP) [6]	$Y_t = t(a + bt + ct^2)^{-1}$
IQP Modified Model [7, 8]	$Y_t^{-1} = a + bt^{-1} + ct + e$
Gamma function [9]	$Y_t = at^b e^{-ct}$; $\ln(Y_t) = \ln(a) + b\ln(t) - ct + e$
Quadratic model [10]	$Y_t = a + bt - ct^2$
Quadratic cum log model [11]	$Y_t = a + bt + ct^2 + d\ln(t) + e$
Wilmink model [12] 4 models with k=0.050, 0.065, 0.610, and 0.100	$Y_t = a + be^{-kt} + ct + e$
Ali and Schaeffer model (ALI) [13]	$Y_t = a + bx + cx^2 + d\log(1/x) + f\log(1/x)^2 + e$
Mixed log model (ML) [14, 15]	$Y_t = a + bt^{1/2} + c\log(t) + e$
Grossman model [16]	$Y_t = at^be^{-ct}(1 + u\sin(x) + v\cos(x))$; $\ln(Y_t) = \ln(a) + b\ln(t) + ct + u\sin x + v\cos x + e$
Cubic model [17]	$Y_t = a + bt + ct^2 + dt^3 + e$

where, Y_t = Daily milk yield, t = days in milk, e = random error, and a, b, c, d, f, u and v - regression coefficients. In Wilmink model, k is a constant term, while in ALI model, x = t /305 and in Grossman model, x = Day of year. ln is natural logarithm to base e.

From the various models used and fitted, the goodness of fit of the models was evaluated using the following criteria:

2.1. Coefficient of determination (R^2) – which gives the percentage of variance of monthly yields explained by the model:

$R^2 = [1 - (SSE/SST)] \times 100$

where, SSE is error sum of square and SST is total sum of square.

2.2. Adjusted R^2 – which is the adjusted square of correlation coefficient between actual milk yield and predicted milk yield according to model:

$R^2_{adj} = 1 - \{[SSE / (n-p-1)] / [SST / (n-1)]\}$

where, SSE is error sum of squares, SST is total sum of squares, n is number of observations, and p is number of parameters in each function.

R^2_{adj} was used in addition to R^2_{adj}, because R^2_{adj} is adjusted for the number of parameters in the model (p) to make accurate comparison of models.

2.3. Root Mean Square Error - is a kind of generalised deviation.

$$RMSE = \sqrt{\frac{[\sum_{i=1}^{n}(y_i - \hat{y})^2]}{n}}$$

where, n is the number of observations, y_is are the actual values and ŷs are values predicted by the regression model. Smaller RMSE would indicate better modelling.

2.4. Durbin-Watson (DW) Coefficient - is given by

$$DW = \frac{\sum_t^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2}$$

where, et is residual at time t and et-1 is residual at time t-1.

The Durbin-Watson statistic tests the null hypothesis that the residuals from an ordinary least-square regression are not auto-correlated against the alternative that the residuals follow an auto-regressive process. The Durbin-Watson statistic ranges from 0 to 4. A value nearer to two indicates non-auto-correlation, a value toward 0 indicates positive auto-correlation and a value toward 4 indicates negative auto-correlation. Friedman's test was used to rank different models according to the mean rank obtained by this test.

All the models used in this study were ranked based on each of the above four diagnostic criteria. The ranks obtained were then analysed using Friedman's test to get the mean rank, based on which the model of best fit was identified.

3. Results

The means ± S.E. of various lactation parameters for Jersey crossbreds are presented in Table 1. The parameter 'a' was positive and ranged between 0.033 (IQP modified model) to 9.958 (IQP model. The parameter ‘b’ was positive in Gamma function and IQP modified model, while it was negative and ranged from -53.125 (Wilmink model, k=0.61) to -0.002 (Parabolic exponential model (PEM). Among the models with more than three parameters, Grossman model had positive values for ‘b’ as 0.117 and in Cubic, ALI and QCL models, the values were -0.023, 5.788 and -0.023, respectively.

Table 1: Parameters estimated of various lactation curve models fitted for Jersey crossbreds

Lactation Curve Model	a	b	c	d	f	u	v
Gamma	1.174 ± 0.022	0.122 ± 0.017	-0.005 ± 0.000				
Wilmink (k=0.050)	8.079 ± 0.162	-0.959 ± 0.863	-0.015 ± 0.003				
Wilmink (k=0.065)	8.104 ± 0.156	-0.295 ± 0.247	0.001 ± 0.025				
Wilmink (k=0.610)	8.046 ± 0.145	-53.125 ± 9.970	-0.018 ± 0.001				
Wilmink (k=0.100)	8.312 ± 0.263	-0.590 ± 0.330	0.006 ± 0.019				
Cubic	8.106 ± 0.155	-0.023 ± 0.005	0.000 ± 0.000	0.000 ± 0.000			
Grossman	7.580 ± 0.464	0.117 ± 0.017	-0.006 ± 0.009			0.003 ± 0.001	0.000 ± 0.001
Inverse quadratic polynomial	9.958 ± 1.047	-0.180 ± 0.030	0.003 ± 0.001				
Inverse quadratic polynomial modified	0.033 ± 0.015	0.916 ± 0.089	0.001 ± 0.000				
Mixed log	6.534 ± 0.301	-0.673 ± 0.034	1.398 ± 0.139				

Ali and Schaeffer	0.442 ± 1.386	5.788 ± 2.299	-3.491 ± 1.161	4.658 ± 0.730	-0.769 ± 0.122		
Quadratic	8.121 ± 0.147	-0.019 ± 0.002	0.000 ± 0.000				
Quadratic cum log	7.632 ± 0.338	-0.023 ± 0.004	0.000 ± 0.000	0.173 ± 0.128			
Exponential decline	8.836 ± 0.168	-0.004 ± 0.000					
Parabolic exponential	7.851 ± 0.158	-0.002 ± 0.000	0.000 ± 0.000				

The parameter ‘c’ in models having three parameters ranged from -0.018 (Wilmink model, k=0.61) to 1.398 (Mixed log model). In ALI, this parameter was -3.491. Wilmink curves were both of standard and continuously increasing types for different values of constant k. Parameters ‘d’ and ‘f’ in ALI, which represent the increasing slope of lactation curve, were 4.658 and -0.769, respectively. Parameters ‘u’ and ‘v’ for Grossman model were 0.003 and 0.000, respectively. Standard Error estimates (Table 1) showed that almost all estimates obtained were precise.

3.1 Best Lactation Curve Model for Jersey Crossbreds
Statistical parameters for identifying the model of best fit for Jersey crossbreds are presented in Table 2. The R^2 and the adjusted R^2 values were the highest in Cubic, IQP and ALI models (0.74), which meant that all these models were able to explain 74 per cent of variation in daily milk yield. Wilmink (k=0.065) and quadratic cum log models also had higher R^2 (0.73). R^2 was the lowest (0.45) in IQP modified model. It can be said that all the models considered for fitting lactation curves had good R^2 values (>0.62), except IQP modified model.

Table 2: Statistical parameters for identifying the best lactation curve model for Jersey crossbreds

Lactation Curve Model	**R^2**	**R^2_{adj}**	**RMSE**	**DW**
Gamma	0.69	0.69	0.20	0.85
Wilmink (k=0.050)	0.70	0.69	0.84	0.91
Wilmink (k=0.065)	0.73	0.68	0.86	0.91
Wilmink (k=0.610)	0.66	0.65	0.90	0.84
Wilmink (k=0.100)	0.69	0.68	0.86	0.89
Cubic	0.74	0.73	0.78	1.03
Grossman	0.67	0.66	0.21	0.85
Inverse quadratic polynomial	0.74	0.74	2.28	0.87
Inverse quadratic polynomial modified	0.45	0.45	0.08	0.76
Mixed log	0.70	0.70	0.84	0.92
Ali and Schaeffer	0.74	0.74	0.77	1.09
Quadratic	0.70	0.70	0.98	0.95
Quadratic cum log	0.73	0.73	0.79	1.03
Exponential decline	0.62	0.61	0.22	0.75
Parabolic exponential	0.68	0.68	0.20	0.90

In Gamma function, R^2 was 0.69, as reported [18]. However, this value obtained in this study was lower than the value reported [19] in half Jersey and half Sahiwal cows. R^2 value obtained in Exponential decline function was 0.62 in this study, which was lower than R^2 value of 0.98 reported [18]. It also needs mention that the R_{adj}^2 values do not differ greatly from their respective R^2.

RMSE values, which are the measures of generalized deviation, in the models fitted ranged from 0.08 kg (IQP modified) to 0.98 kg (Quadratic model), except the IQP model which gave an exceptional higher RMSE value of 2.28 kg. Durbin Watson (DW) coefficients in the models fitted ranged from 0.76 (IQP modified) to 1.09 (ALI) which indicated that all the models had positive auto- correlation of residuals.

Based on these four diagnostic parameters, the model of best fit was obtained by using the coefficient obtained from Friedman’s rank test. Ranking of models based on mean ranks for Jersey crossbreds is presented in Table 3. Friedman’s rank test showed that difference in mean ranks was significant (p<0.05). The best three models that could be chosen for prediction of milk yield in Jersey crossbreds were ALI, Cubic and QCL, based on the order of ranks obtained.

Table 3: Mean ranks of various lactation curve models fitted for Jersey crossbreds

Lactation Curve Model	**Mean Rank**
Ali and Schaeffer	13.50
Cubic	12.25
Quadratic cum log	11.25
Quadratic	9.25
Mixed log	9.00
Inverse quadratic polynomial	8.75
Wilmink (k=0.050)	8.25
Parabolic exponential	8.00
Gamma	7.88
Wilmink (k=0.065)	7.75
Grossman	6.25
Wilmink (k=0.100)	6.13
Inverse quadratic polynomial modified	4.75
Exponential decline	4.00
Wilmink (k=0.61)	3.00

(Chi-Square = 24.067**, p<0.05)

However, polynomial function of degree 5 was the model of best fit for Jersey crossbreds[18]. Yet, IQP model was the best function in explaining the first lactation curve based on monthly as well as weekly milk records of Jersey x Sahiwal F1 cows [19]. These differences could be due to variations in the source and nature of data included in the analysis.

4. Discussions
The positive parameter 'a' in all models clearly indicated that this parameter explained the increasing part of the lactation curve. Among the models with more than three parameters, Grossman model had positive values for ‘b’ as 0.117 and in Cubic, ALI and QCL models, the values were -0.023, 5.788 and -0.023, respectively. The parameter ‘c’ in models having three parameters ranged from -0.018 (Wilmink model, k=0.61) to 1.398 (Mixed log model). In ALI, this parameter was -3.491.

The classification of lactation curves based on the sign of parameters 'b' and 'c' of Gamma function and Wilmink model revealed that the Gamma curves were typical standard curve for Jersey crossbreds. Standard errors in all models showed that almost all estimates that were obtained were precise.

The highest R^2 and adjusted R^2 values in Cubic, IQP and ALI models indicated that all these models could explain 74 per cent of variation in daily milk yield. Wilmink (k=0.065) and quadratic cum log models also had higher R^2 (0.73). However, the lower R^2 (0.45) in IQP modified model implied that it is a poor model. Otherwise all the other models considered for fitting lactation curves had good explanatory power, as indicated by higher R^2 values (>0.62).

In Gamma function, R^2 was 0.69, as reported [20]. However, this value obtained in this study was lower than the value reported [21] in half Jersey and half Sahiwal cows. R^2 value obtained in Exponential decline function was 0.62 in this study, which was lower than R^2 value of 0.98 reported[20]. It also needs mention that the R_{adj}^2 values do not differ greatly from their respective R^2.

RMSE values, which are the measures of generalized deviation, in the models fitted ranged from 0.08 kg (IQP modified) to 0.98 kg (Quadratic model), except the IQP model which gave an exceptional higher RMSE value of 2.28 kg. Durbin Watson (DW) coefficients in the models fitted ranged from 0.76 (IQP modified) to 1.09 (ALI) which indicated that all the models had positive auto- correlation of residuals.

Based on R^2, R_{adj}^2, RMSE and DW values, and the Friedman's rank test, the best three models that could be chosen for prediction of milk yield in Jersey crossbreds were ALI, Cubic and QCL. However, polynomial function of degree 5 was the model of best fit for Jersey crossbreds[20]. Yet, IQP model was the best function in explaining the first lactation curve based on monthly as well as weekly milk records of Jersey x Sahiwal F1 cows [21].

5. Conclusion

The highest R^2 (0.74) was obtained in Cubic, IQP and ALI models, with the lowest (0.45) in IQP modified model, while RMSE was the lowest in IQP modified model and highest in Quadratic model. DW coefficients obtained indicated the positive auto-correlation of residuals in all the models fitted. Based on these, the best lactation curve models for Jersey crossbreds were found to be ALI, Cubic and QCL models. Thus, these models can be of use for best prediction of daily milk yield in Jersey crossbreds. Also, the lactation parameters estimated through this study can be used for genetic evaluations and for selection of cows to improve persistency and quantum of milk yield.

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